

Asymptotic Series and the Method of Vanishing Viscosity

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Introduction. We consider the following second order elliptic quasilinear partial differential equation (PDE)

$$(0.1)_\varepsilon \quad \begin{cases} -\varepsilon \Delta u^\varepsilon + H(x, Du^\varepsilon) + \lambda u^\varepsilon = 0 & \text{for } x \in \Omega, \\ u^\varepsilon(x) = 0 & \text{for } x \in \partial\Omega, \end{cases}$$

where $\varepsilon > 0$ is a small parameter, Ω is a bounded open subset of \mathbf{R}^N with smooth boundary $\partial\Omega$, $\lambda \geq 0$, H is a smooth function of its arguments, and Du denotes the gradient of u . We are interested in the case where ε is small and want to express u^ε in terms of the solution u^0 of the first order PDE

$$(0.1)_0 \quad \begin{cases} H(x, Du^0) + \lambda u^0 = 0 & \text{for } x \in \Omega, \\ u^0(x) = 0 & \text{for } x \in \partial\Omega. \end{cases}$$

We continue by stating the assumptions we make. In what follows we denote by H_x , H_p the gradients of $H(x, p)$ in the variables x , p , by H_{pp} the matrix of second order partial derivatives $H_{p_i p_j}$, $i, j = 1, \dots, N$, and by H^+ the positive part of H . We assume:

- (A₁) Ω is open and bounded with C^∞ boundary $\partial\Omega$.
- (A₂) $\lambda \geq 0$.
- (A₃) $H(x, p)$ is C^∞ and $H_{pp} > 0$.
- (A₄) $H(x, 0) \leq -\alpha < 0$.
- (A₅) $\frac{H(x, p)}{|p|} \rightarrow \infty$ as $|p| \rightarrow \infty$ uniformly in x .
- (A₆) $|H_x| \leq K[(p \cdot H_p - H) + |p| + 1 + H^+]$, where K is a nonnegative constant.