

A Strong Approximation Theorem for Real Cycles

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We will use the notation and terminology of [AF1]. Our main result, which answers a question posed by Almgren concerning the isoperimetric inequality (see the Remark at the end of this paper), is the following.

Theorem. *Corresponding to each size bounded rectifiable cycle $T = \mathbf{t}(S, \vartheta, \xi) \in \mathbf{S}_m(\mathbf{R}^{m+n})$ and each $\varepsilon > 0$ there exists a bi-lipschitzian diffeomorphism $f: \mathbf{R}^{m+n} \rightarrow \mathbf{R}^{m+n}$ and a polyhedral cycle $P \in \mathbf{P}_m(\mathbf{R}^{m+n})$ with the following properties:*

- (1) $\text{Lip}(f) < 1 + \varepsilon, \text{Lip}(f^{-1}) < 1 + \varepsilon.$
- (2) $\text{spt}(f) \subset \{x: \text{dist}(x, \text{spt}(T)) < \varepsilon\},$
- (3) $|f(x) - x| < \varepsilon$ for each $x \in \mathbf{R}^{m+n},$
- (4) $\mathbf{M}(T - f_{\#}P) < \varepsilon, \mathbf{S}(f_{\#}P) < (1 + \varepsilon)\mathbf{S}(T),$
- (5) $\Theta^m(\|P\|, x)$ is a rational number for $\|P\|$ almost every $x \in \mathbf{R}^{m+n}.$

Proof. Applying the Strong Approximation Theorem of [AF1; 2.11] to a cone over T and considering the boundary of the resulting polyhedral chain, we see that we obtain a bi-lipschitzian diffeomorphism f and a polyhedral cycle $Q \in \mathbf{P}_m(\mathbf{R}^{m+n})$ satisfying (1)–(4) with P replaced by Q . Suppose Q is of the form

$$Q = \sum_{j=1,2,\dots,N} X_j \Xi_j,$$

where the Ξ_j are m -dimensional simplices which are \mathcal{H}^m almost pairwise disjoint and the X_j are non-zero real numbers. Let $\Delta_1, \Delta_2, \dots, \Delta_M$ be a set of $(m-1)$ -dimensional simplices which are \mathcal{H}^{m-1} almost pairwise disjoint and are such that