

# *Bubble Growth in Porous Media*

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**§0. Introduction.** Consider  $\mathbf{R}^N$  as a homogeneous porous medium filled with viscous fluid. A light fluid, say air, is injected. It forms a bubble which occupies space  $D(t)$  at each time  $t$ . Fluid is withdrawn at infinity at a total rate  $Q$ . As a result, the bubble  $D(t)$  increases with time. The model represents an ill posed free boundary problem; for  $N = 2$  it coincides mathematically with the flow describing the Hele–Shaw cell.

The main question is how to determine those initial positions  $D(0)$  of the bubble for which a solution will exist for all  $t > 0$ . The answer should evidently depend on the final position  $D(\infty)$  of the bubble, or on the set  $\Sigma = \mathbf{R}^N \setminus D(\infty)$  which represents the fingers produced by the limiting phase of the expanding bubble.

In this paper we prescribe the set  $\Sigma$  and determine whether or not there exists, in fact, a solution  $D(t)$  with  $D(\infty) = \mathbf{R}^N \setminus \Sigma$ ; by a solution we shall usually understand some kind of a weak solution. In the case where  $\Sigma$  is empty we prove that the class of all possible solutions, either weak or classical, coincide with those given by ellipsoids

$$(0.1) \quad D(t): \sum_{i=1}^N \frac{x_i^2}{a_i^2} \leq \lambda^2(t)$$

with  $\text{meas } D(t) = \text{meas } D(0) + Q(t)$ , i.e.,

$$(0.2) \quad \lambda^N(t) = \frac{Q}{\mu} t + 1 \quad (\mu = \text{meas } D(0)).$$

Suppose  $\Sigma \neq \emptyset$ , and take for example  $\Sigma$  to consist of several pairs of fingers; say  $E_i$  is a domain in  $\mathbf{R}^{N-1}$  and  $\Sigma$  is made up of

$$(0.3) \quad \{(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_N) \in h_i(|X_i|), |x_i| > a_i\}$$