

On the Dirichlet Problem with L^2 Boundary Values in a Half-space

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§1. Introduction. In this article we consider the Dirichlet problem with L^2 boundary data for a linear second-order uniformly elliptic divergence-structure operator in a half-space. This problem has been studied recently by the first author [2], [3] under various differentiability hypotheses on some of the principal coefficients of the operator using an energy inequality method. The main purpose of this work is to show that this method can be used when these hypotheses are relaxed to ones of continuity at the boundary: with a Dini-type modulus in the normal direction and with exactly a Dini modulus in the tangential directions. In addition, we weaken considerably the hypotheses on the lower order coefficients. We mention that Fabes, Jerison, and Kenig [5] have also studied this problem under somewhat different hypotheses using much deeper methods. They considered the same modulus of continuity in the normal direction and an arbitrary one in the tangential directions (but on *all* the principal coefficients), and their lower order coefficients were all assumed to be zero. More recently Dahlberg [4] has obtained results without any continuity in the tangential directions using methods similar to those in [5].

This paper is organized as follows. Section 2 contains the definitions, assumptions, and preliminary results to be used. Section 3 discusses the existence of a trace on the boundary of a half-space for solutions of the differential equation in question. In particular, Theorem 3 shows that the trace exists when the solution belongs to the function space $\widetilde{W}^{1,2}$ defined in Section 2. Thus, Theorem 3 justifies the introduction of this function space and the present approach to the Dirichlet problem. Finally, Section 4 is devoted to proving the existence of a solution to the Dirichlet problem under appropriate conditions.

§2. Preliminaries. For $n \geq 3$ we denote by $x = (x_1, \dots, x_n)$ a point in \mathbf{R}_n , we identify \mathbf{R}_n with $\mathbf{R}_{n-1} \times \mathbf{R}$ in the usual way by writing $x = (x', x_n)$ with