

Smooth \mathbf{Z}_p Actions on $S^n \times D^k$

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§0. Introduction. Suppose that G is a finite group with periodic cohomology of period r acting smoothly and semifreely on a homotopy sphere Σ^{n+k} with fixed set F^n . If $F_1^n \subset F^n$ is a submanifold with boundary having the $\mathbf{Z}_{(P)}$ homology of S^m ($0 < m < n$) (P is the set of primes dividing $|G|$), then F_1^n determines an element in $\tilde{K}_0(\mathbf{Z}G)$ defined by $\tilde{\chi}(F_1) = \sum_{i \geq 0} (-1)^i [\text{Tor } H_i(F_1)]$. In [5] we obtained the following:

Theorem. *If G , Σ^{n+k} , and F_1 are as above and $n+k \geq 6$, $k > n+2$, then there exists a G -invariant $S^n \times D^k$ in Σ^{n+k} with fixed set F_1 if and only if $\tilde{\chi}(F_1) = 0$ in $\tilde{K}_0(\mathbf{Z}G)$.* □

As a result there is an equivariant splitting of

$$\Sigma^{n+k} = S^n \times D^k \cup D^{n+1} \times S^{k-1}$$

corresponding to the splitting of $F = F_1 \cup \text{cl}(F - F_1)$, when $\tilde{\chi}(F_1) = 0$.

In the present paper we restrict our attention to $G = \mathbf{Z}_p$ (p odd, prime) actions on $S^n \times D^k$. Suppose G acts smoothly on $S^n \times D^k$ with fixed set F^n , an n -manifold with boundary and a $\mathbf{Z}_{(p)}$ -homology m -sphere ($0 < m < n$). We determine conditions under which there exists a G -invariant homotopy n -sphere $\Sigma^n \subseteq S^n \times D^k$ (with nonzero degree (mod p)) such that Σ^G is a homotopy m -sphere $\Sigma^m \subseteq F^n$ (with nonzero degree (mod p)). Denote by ν the normal bundle to F^n in $S^n \times D^k$.

Theorem. *Suppose $G = \mathbf{Z}_p$ (p odd prime) acts smoothly on $S^n \times D^k$ with fixed set F^n a manifold with boundary having the $\mathbf{Z}_{(p)}$ -homology of S^m ($0 < m < n$). Assume that:*

- (1) $m \geq 5$, $k > n+2$, $n > 2m+1$,
- (2) m odd (which implies n odd),
- (3) $p \geq (n+3)/2$,
- (4) F is $[\frac{m}{2}]$ -connected and stably parallelisable (which implies that ν is trivial).