

# *Holomorphic Sections of a Homogeneous Line Bundle Associated to an Infinite-Dimensional Lie Group*

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**Introduction.** The first part of this article deals with a certain type of analytic structure on a particular Banach manifold. Fix a positive integer  $n$ , and let  $z$  be an indeterminate. Define a group  $G^c$  by

$$G^c \equiv \left\{ g = \sum_{k=-\infty}^{\infty} A_k z^k \mid A_k \in M_n(\mathbb{C}) \text{ for all } k, \right. \\ \left. \sum_{k=-\infty}^{\infty} |A_k| < \infty, \det g = 1 \right\},$$

where  $|A_k|$  denotes the usual norm of an element of  $M_n(\mathbb{C})$ , the space of  $n \times n$  matrices over  $\mathbb{C}$ ; and “ $\det g$ ” is defined below. This is, in fact, an infinite-dimensional Lie group. Let  $P^c$  denote the subgroup of elements which only involve terms of nonpositive degree. Then set  $X^c \equiv G^c/P^c$ . We describe a special analytic structure on  $X^c$ , and examine the relationship of  $X^c$  to a dense subset  $X$  consisting of those elements with representatives in a subgroup  $G$  of  $G^c$ ; the subgroup  $G$  consists of those elements that are finite Laurent polynomials with coefficients in  $M_n(\mathbb{C})$ . This is related to work of Pressley [12] and more recently Atiyah and Pressley [1].

The second part of this article deals with a result of Kac and Peterson realizing a certain set of sections of a line bundle over  $X$  as an irreducible module of a central extension of  $G$  (see [7]). We show the same module can be realized as a set of sections of a line bundle over  $X^c$  that are holomorphic with respect to the aforementioned analytic structure. This result suggests an approach to a Borel–Weil theorem for infinite-dimensional homogeneous spaces along the lines of Kostant’s work in [9].

**Notation.** Let  $n$  be a nonzero element of  $\mathbf{Z}^+$ , the non-negative integers, throughout this article. We denote the identity element of  $GL_n(\mathbb{C})$ , as well as that of  $G$ , by  $I$ .