

# *Characterizing Blowup Using Similarity Variables*

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**§1. Introduction.** We are interested in the blowup of solutions of the semilinear parabolic equation

$$(1.1) \quad \begin{aligned} u_t - \Delta u - |u|^{p-1}u &= 0 && \text{in } D \times (0, T) \\ u &= 0 && \text{on } \partial D \times (0, T), \end{aligned}$$

in  $n$  space dimensions, with a subcritical nonlinearity

$$(1.2) \quad 1 < p < \frac{n+2}{n-2} \quad \text{or} \quad n \leq 2.$$

Our principal results are a bound on the growth rate, and a characterization of the asymptotic behavior of  $u$  near blowup. More general semilinear equations  $u_t - \Delta u - f(u) = 0$  are considered in §6, where we establish the same results whenever  $f(u) - |u|^{p-1}u$  grows at most as  $|u|^q$  for some  $q < p$ . In the final section, which can be read independently of the others, we apply these results to prove new lower bounds on the blowup rates of certain norms of  $u$ .

Our first goal is the growth rate estimate

$$(1.3) \quad \sup_x |u(x, t)| \leq C(T - t)^{-1/(p-1)},$$

where  $T$  is the maximal existence time of the  $L^\infty$  solution  $u$ . It asserts that  $u$  blows up at the same rate as the solution of the ordinary differential equation  $v_t - |v|^{p-1}v = 0$ ,

$$v(t) = \pm k(T - t)^{-1/(p-1)}, \quad k = (p - 1)^{-1/(p-1)}.$$

Local-in-time existence theory gives the opposite inequality. The bound (1.3) was first proved by Weissler in [40], for radial solutions on a ball with rather special initial data. Friedman and McLeod subsequently proved it in [12] for solutions on a bounded, convex domain, with initial data satisfying

$$(1.4) \quad u_0 \geq 0, \quad \Delta u_0 + u_0^p \geq 0.$$