

On Ahlfors' Spiral Generalization of the Denjoy Conjecture

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§1. In his thesis [1] Ahlfors not only proved the Denjoy conjecture but presented a version of it generalized to the case where the asymptotic paths satisfy a certain condition of increase of argument. In the commentary contained in his *Collected Papers* [2] Ahlfors remarks that his proof for the latter result is inadequate but that an unpublished proof has been given by Hayman by quite different techniques. Walter Hayman tells me that his proof is quite long and arduous so it seems to be of some interest to show that it is possible to prove the result in a fairly straightforward manner by the method of the extremal metric in the context of my earlier proofs of the Denjoy conjecture [3, 4].

§2. We begin by recalling several basic concepts and results.

By a triad (D, α, P) we mean the configuration consisting of a simply-connected domain D of hyperbolic type, an open border arc α of D and a point P interior to D . By the module $m(P, \alpha, D)$ we mean the module of the family of locally rectifiable open arcs in $D - \{P\}$ running from α back to α and separating P from the closed border arc α^* of D complementary to α .

For real numbers a, b , $a < b$, we denote by $S(a, b)$ the strip in the (u, v) -plane defined by $a < u < b$ and by $g(a)$ its open border arc determined by the line $u = a$.

Let (D, α, P) be a triad in the z -sphere, with D bounded by a Jordan curve so that α, α^* can be regarded as the corresponding boundary arcs, $u(z)$ a function subharmonic in D , a, b real numbers with $a < b$, such that

$$\overline{\lim}_{z \rightarrow \zeta} u \leq a, \quad \zeta \in \alpha, \quad \overline{\lim}_{z \rightarrow \zeta} u(z) \leq b, \quad \zeta \in \alpha^*,$$

and

$$u(P) > a.$$

Then

$$m(P, \alpha, D) \leq m(u(P), g(a), S(a, b)).$$