

Bloch Functions, BMO, and Boundary Zeros

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§1. Introduction. Throughout this paper, n will be an integer, $n > 1$; the letters U and B will denote the open unit disc in \mathbf{C} and the open euclidean unit ball in \mathbf{C}^n , respectively. In [1] it was shown that the monomial

$$(1) \quad \pi(z) = n^{n/2} z_1 z_2 \cdots z_n,$$

which maps B onto U has the following pullback property:

If g is holomorphic in U and belongs to the Bloch space, then $g \circ \pi$ is in BMOA.

(The relevant definitions will be given in §III.)

A similar result was proved in [1] in the context of H^p -functions. Paula Russo [13], [14] then showed that analogous pullback theorems hold for the map $\pi: B \rightarrow U$ defined by

$$(2) \quad \pi(z) = z_1^2 + \cdots + z_n^2.$$

Our work on the present paper began with the observation that the results mentioned above lead to new information about the boundary values of holomorphic functions in B . For example (see §§VI and VII):

There is an $f \in \text{BMOA}$ (and even in VMOA), not identically 0, with the following property: If $h \in H^\infty(B)$ and if the radial limits of h are 0 at every boundary point where those of f are 0, then $h \equiv 0$.

The same is true with $H^p(B)$ and $H^q(B)$ in place of BMOA and $H^\infty(B)$, whenever $0 < p < q$.

Section VI contains a gap theorem for $H^1(B)$ which seems to have no analogue in one variable. Here is a special case of it, for $n = 2$, and π as in (1):