

Completely Positive Maps and Approximate Equivalence

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§1. Introduction. In [H3] the author extended D. Voiculescu's theorem [V, Theorem 1.5] on approximate equivalence to arbitrary representations of arbitrary C^* -algebras. In addition, [H3] contains an analogue of Voiculescu's theorem for approximate subrepresentations. The main idea of the present paper is to combine Voiculescu's theorem with W. F. Stinespring's theorem [St] on completely positive maps to prove an analogue of Voiculescu's theorem for approximate compressions of a representation. The results on approximate compressions extend J. Glimm's characterization [G] of the weak $*$ limits of vector states of a C^* -algebra of operators to completely positive maps that are limits of spatial completely positive maps. Glimm's result [G] has already been extended to matrix-valued completely positive maps by J. Bunce and N. Salinas [BS], and partially extended to operator-valued completely positive maps by W. Arveson [Ar3, Thm. 4].

Our results on approximate compressions and summands (as well as Voiculescu's theorem) have analogues for arbitrary operator-valued functions on some fixed set (not just representations or completely positive maps on a C^* -algebra). Furthermore, for functions there is a notion of approximate restriction that (unlike the case of self-adjoint maps) is distinct from the notion of approximate summand.

The results on approximate summands, restrictions and compressions lead naturally to operator-valued analogues of reducing spectrum, approximate point spectrum, and closed numerical range for operators on a separable Hilbert space. We show that when T is an operator on a separable infinite-dimensional space, the closures of the unitary equivalence class of T in the $*$ -strong, strong, and weak operator topologies behave like operator-valued analogues of the reducing spectrum, approximate point spectrum, and closed numerical range of T , respectively. The results on operator-valued spectra easily extend to joint spectra, which in turn leads to an extension of the results in [H2] to a theory of decomposable functions of several variables.