

Hopf Bifurcation for Nonlinear Integrodifferential Equations in Banach Spaces with Infinite Delay

GIUSEPPE DA PRATO & ALESSANDRA LUNARDI

§0. Introduction. We are here concerned with periodic solutions of some fully nonlinear integrodifferential equations in a Banach space X :

$$(0.1) \quad u'(t) = f(\lambda, u(t)) + \int_{-\infty}^t g(\lambda, t-s, u(s)) ds,$$

where λ is a real parameter, $f:]-1, 1[\times D \rightarrow X$, $g:]-1, 1[\times]0, +\infty[\times D \rightarrow X$ are C^∞ functions, D being continuously imbedded in X .

Our results cover, for instance, the following example:

$$(0.2) \quad \left\{ \begin{array}{l} u_t(t, x) = \varphi(\lambda, u(t, x), u_x(t, x), u_{xx}(t, x)) \\ \quad + \int_{-\infty}^t \psi(\lambda, t-s, u(s, x), u_x(s, x), u_{xx}(s, x)) ds, \\ \quad \quad \quad 0 \leq x \leq \pi \\ u(t, 0) = u(t, \pi) = 0, \end{array} \right.$$

where $X = C([0, \pi])$, $D = C_0^2([0, \pi])$, φ and ψ are smooth functions.

Problems of this kind have been widely studied in the finite-dimensional case (see for instance [S], [C1], [C2]). Concerning the biological relevance of periodic solutions of equation (0.1), see [C1], [C2].

In the infinite-dimensional case we only know two papers on the subject: [NY] and [T]. In [NY] the authors study an equation consisting of a linear term with infinite delay and a nonlinear term with finite delay. In [T] a special case has been treated in which the nonlinear term is of the form

$$\int_{-\infty}^t g(\lambda, t-s, u(t), u(s)) ds$$

with $g:]-1, 1[\times]0, +\infty[\times X^2 \rightarrow X$.