

Blowup, Compactness and Partial Regularity in the Calculus of Variations

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§1. Introduction. In [7] the first author proved partial regularity of minimizers for certain problems in the calculus of variations, described as follows. Let n, N denote positive integers, write $M^{n \times N}$ for the space of all real $n \times N$ matrices, and assume $\Omega \subset \mathbf{R}^n$ is open, bounded and smooth. Then for functions $v: \Omega \rightarrow \mathbf{R}^N$ consider the functional

$$(1.1) \quad I[v] \equiv \int_{\Omega} F(Dv) \, dx,$$

where

$$Dv \equiv \left(\left(\frac{\partial v^i}{\partial x_{\alpha}} \right) \right) \quad (1 \leq \alpha \leq n, \quad 1 \leq i \leq N)$$

is the gradient matrix of v and

$$F: M^{n \times N} \rightarrow \mathbf{R}$$

is given.

We are interested in the possible regularity of minimizers of $I[\cdot]$ among all appropriate functions subject to some given, but here unspecified, boundary conditions. The main result of [7] asserts that any such minimizer is in fact continuously differentiable on an open subset $\Omega_0 \subset \Omega$, with $|\Omega - \Omega_0| = 0$; this [7] proves under the primary assumption that F be *uniformly strictly quasiconvex*, which means

$$(1.2) \quad \int_0^1 F(A) + \gamma |D\varphi|^2 \, dx \leq \int_0^1 F(A + D\varphi) \, dx$$

for some $\gamma > 0$ and all open $0 \subset \mathbf{R}^N$, $A \in M^{n \times N}$, $\varphi \in C_0^1(0; \mathbf{R}^N)$. This is interesting since quasiconvexity (i.e., condition (1.2) with $\gamma = 0$) is both necessary and sufficient for the weak sequential lower semicontinuity of $I[\cdot]$ on appropriate Sobolev spaces, and is consequently a primary hypothesis in the