

Pointwise A Priori Bounds for Strongly Coupled Semilinear Parabolic Systems

REINHARD REDLINGER

*Dedicated to Professor Wolfgang Walter
on the occasion of his sixtieth birthday*

§1. Introduction. In this paper we consider semilinear parabolic boundary value problems of the form

$$\frac{\partial u}{\partial t} + L(t, x)u = f(t, x, u, Du, \dots, D^{2m-1}u) \quad \text{in } J \times \Omega, \quad (1)$$

$$B(t, x)u = 0 \quad \text{on } J \times \partial\Omega,$$

where Ω is a bounded domain in \mathbf{R}^n , $J = (0, T)$ with $T \leq \infty$, and L is a strongly coupled, uniformly elliptic operator of order $2m$ acting on N -vector-valued functions u . By B we denote a homogeneous, possibly time-dependent linear boundary operator for $m = 1$, and the Dirichlet boundary operator B_D for $m > 1$. It is the purpose of this paper to establish pointwise a priori bounds for solutions of (1). The same problem in case $m = 1$, $B = B_D$ and $f = f(t, x, u)$ has been previously treated by Cosner [6, 7] and Alikakos [2]. (In [2] and [6] only time-independent operators L and nonlinearities $f = f(x, u)$ are considered.) In these articles the a priori estimates are obtained by first getting a bound on the $W^{1,2}$ -norm of the solution via a Ljapunov function and then using regularity results to improve the bound to a pointwise one. We use the same approach, but with a Ljapunov function that only yields a bound in L^2 . This makes it possible to considerably weaken the structure conditions imposed on the system, but necessitates a more restrictive condition on the growth of the nonlinearity. Thus, in contrast to [2, 6, 7] we do not suppose that f has gradient structure or that L is self-adjoint.

The present paper makes essential use of results and methods developed by Amann in [4, esp. §§6, 7]. In this paper, global existence theorems for classical solutions of the boundary value problem (1) are established. The structure conditions on L and f in [4, §7] are analogous to those imposed by Cosner and Alikakos.