

*Extension of Kerzman's Theorem on  
Differentiability of  
the Bergman Kernel Function*

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In 1972 Kerzman proved that the Bergman kernel function for a smooth bounded strongly pseudoconvex domain in  $\mathbf{C}^n$  is smooth up to the boundary off the boundary diagonal [11]. His proof does not use the full force of the strong pseudoconvexity, being based on a certain pseudolocal estimate from Kohn's theory [12] of the  $\bar{\partial}$ -Neumann problem.

Recent applications of the Bergman kernel function to boundary smoothness of holomorphic mappings (see [3, 8] and their references) have made it important to know the weakest hypotheses that imply local boundary smoothness of the kernel function. Sufficient conditions that constrain only a small portion of the boundary have not previously been well known, partly because the pseudolocal estimate used in the proof seems not to be stated explicitly in Kohn's papers.

The purpose of this note is to prove the following two generalizations of Kerzman's theorem, which are the strongest results currently accessible by essentially his technique. Steve Bell [2] independently has found a different proof of these results by relating the Bergman kernel function of the domain to the Bergman kernel function of a small internal comparison domain.

**Theorem 1.** *Let  $\Omega$  be a smooth bounded pseudoconvex domain in  $\mathbf{C}^n$ . Let  $p$  and  $q$  be two distinct points in the boundary  $b\Omega$ , both of finite type in the sense of D'Angelo [7]. Then there are neighborhoods  $U$  of  $p$  and  $V$  of  $q$  such that the Bergman kernel function extends smoothly to  $(U \cap \bar{\Omega}) \times (V \cap \bar{\Omega})$ .*

**Theorem 2.** *Let  $\Omega$  be a smooth bounded pseudoconvex domain in  $\mathbf{C}^n$ . Suppose the Bergman projection maps the space  $C^\infty(\bar{\Omega})$  into itself. (This is condition R of Bell and Ligočka [4]). If  $p$  is a point of finite type in  $b\Omega$  and  $q$  is any other point of  $b\Omega$ , then there are neighborhoods  $U$  of  $p$  and  $V$  of  $q$  such that the Bergman kernel function extends smoothly to  $(U \cap \bar{\Omega}) \times (V \cap \bar{\Omega})$ .*

The striking point of the first theorem is that it contains no global regularity hypothesis. On the other hand, in the second theorem one variable is allowed to approach a completely arbitrary boundary point.