

The Space of m -dimensional Surfaces That Are Stationary for a Parametric Elliptic Functional

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Introduction. In this paper we study the space of m -dimensional regular immersed surfaces in \mathbf{R}^n that are stationary with respect to m -dimensional area or, more generally, with respect to a parametric elliptic integrand Φ . This includes, for example, the case of surfaces of constant mean curvature c with respect to a Riemannian metric. Let M be a compact connected m -dimensional manifold with nonempty boundary. We regard two maps $f_1, f_2: M \rightarrow \mathbf{R}^n$ as equivalent (i. e., representing the same surface) if $f_1 = f_2 \circ u$ for some diffeomorphism $u: M \rightarrow M$ such that $u(x) = x$ for all $x \in \partial M$. Let \mathcal{M} be the set of equivalence classes $[f]$ of $C^{j,\alpha}$ Φ -stationary immersions $f: M \rightarrow \mathbf{R}^n$. We prove (Theorem 3.3):

- (1) \mathcal{M} is a Banach manifold.
- (2) The projection $\Pi: \mathcal{M} \rightarrow C^{j,\alpha}(\partial M, \mathbf{R}^n)$ defined by

$$\Pi([f]) = f|_{\partial M}$$

is a smooth map of Fredholm index 0. (That is, the derivative of Π has a finite-dimensional kernel and a closed image of the same finite codimension.)

- (3) Π is a diffeomorphism near $[f]$ if and only if f has no non-tangential Jacobi fields that vanish on ∂M . (“Jacobi field” means “solution to the linearized Euler-Lagrange system”; see 1.4.)

We use these results to study generic properties of stationary surfaces. One useful notion of genericity in infinite-dimensional spaces is provided by Baire category. Frank Morgan [M1, M2, M3] introduced another by defining a measure on $C^{j,\alpha}(\partial M, \mathbf{R}^n)$. In this paper we give a third by defining what it means for a subset of a Banach space to have codimension c (≥ 1). This notion is stronger than the other two in that any set of codimension ≥ 1 must be exceptional in the sense of Baire and also must have measure 0 according to Morgan’s definition.

It follows immediately from (1), (2), and (3) and Smale’s infinite dimensional version of Sard’s theorem that for almost all (in the sense of Baire) $\gamma \in C^{j,\alpha}(\partial M, \mathbf{R}^n)$, $[f] \in \Pi^{-1}(\gamma)$ implies that $[f]$ has no non-tangential