

On a Local Nonlinear Commutant Lifting Theorem

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§1. Introduction. Recently there has been a surge of renewed interest in complex interpolation theory. From the strictly mathematical point of view, it has been found that operator theory can provide far-reaching and powerful generalizations of classical Nevanlinna–Pick interpolation (see e.g. the work of Adamjan–Arov–Krein [1], [2], Ball–Helton [4], Sarason [19], and Sz. Nagy–Foias [20], [21]).

However, a strong motivating factor in this new research in (generalized) interpolation theory is its widespread and manifold applications, especially in engineering. Indeed, these methods have appeared in control and systems theory to solve the H^∞ -optimal weighted sensitivity problem [25], [26], [9], in the area of model reduction [5], [11], and for robust design and stabilization [13], [15], [23]. (See [10] for a much more extensive list of references on the uses of interpolation theory in control.) In circuit theory, there have been applications of Nevanlinna–Pick interpolation to the broadband matching problem [24], [12], and these methods have been even used in studying certain problems in geophysics [8] and biomathematics [7].

Now Sarason [19] (in a special case) and Sz. Nagy–Foias [20] (in complete generality) proved an extremely broad interpolation theorem which has come to be known as the “commutant lifting theorem”. More precisely, let $T:H \rightarrow H$, and $T':H' \rightarrow H'$ be contractions on the Hilbert spaces H , H' . Let $U:K \rightarrow K$, $U':K' \rightarrow K'$ be the minimal isometric dilations T and T' , respectively. (This means that U is such that $P_H U = T P_H$ where $P_H:K \rightarrow H$ is projection, $K = \bigvee_{n=0}^{\infty} U^n H$, and similarly for U' .) Then the commutant lifting theorem [21] asserts that if $A:H \rightarrow H'$ is a contraction intertwining T and T' (i.e., $T'A = AT$) then there exists a contraction $\hat{A}:K \rightarrow K'$ such that $U'\hat{A} = \hat{A}U$, and $P_{H'}\hat{A} = AP_H$. Moreover, \hat{A} may be chosen to be such that $\|\hat{A}\| = \|A\|$. (For the relation of this result see Nevanlinna–Pick interpolation, see [8], [19].)