

Nonlinear Steady Waves in Planetary Vortex Dynamics

BRUCE TURKINGTON

Introduction. In this paper we introduce a class of nonlinear, nonlocal evolution equations in two-space dimensions, and we study steady translational waves governed by these equations. The class that we consider constitutes a mathematical generalization of a standard equation arising in geophysical fluid dynamics, and it is this special case that motivates our general theory. In the terminology of the geophysical applications, we are concerned with planetary vortex dynamics and, in particular, with nonlinear planetary (or Rossby) waves in a nonuniform (sheared) zonal current. The governing equation is given by the so-called β -plane model which provides an approximation to the motion of a layer of ideal fluid (an ocean or atmosphere) on a rotating sphere (a planet) at appropriate length and velocity scales (a quasigeostrophic flow). This equation is an evolution equation for the relative vorticity, and, in essence, it specifies that the sum of the relative vorticity and the ambient vorticity of rotation (the Coriolis parameter) be advected by the flow; it is commonly referred to as the barotropic, quasi-geostrophic (potential) vorticity equation.

Linear planetary waves have been extensively investigated in the geophysical literature, and they are regarded as important features of the large-scale motions of an ocean or atmosphere. A comprehensive treatment of the known linear theory is available in Pedlosky [13]. As pointed out originally by Rossby, the existence of these waves is associated with the latitudinal variation of the Coriolis parameter.

Nonlinear planetary waves in a zonal current owe their existence to an additional effect: the nonuniformity of the undisturbed parallel flow (zonal current). Recognizing this, Long [11] has constructed solitary wave disturbances of a nonuniform zonal current using formal asymptotic methods appropriate to weakly nonlinear long waves. In the same context, Benney [3, 4] and Redekopp [14] have derived the Korteweg-deVries equation as the governing evolution equation for the long-wave approximation. Steady translational waves are known to be present therefore in the form of both solitary and (long-periodic) cnoidal waves, at least in this approximate description. The relevance of these and other nonlinear wave and eddy motions to the modeling of observed, long-lived coherent structures in large-scale geophysical fluid flows (the Great Red