

# Uniqueness and Nonuniqueness of the Plateau Problem

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**§1. Introduction.** The aim of this note is twofold. First we want to show the following:

**Theorem.** *Let  $\Omega \subset \mathbf{R}^n$ ,  $n \geq 2$ , be a bounded  $C^1$ -domain and let  $\Omega_\delta = \{x \in \Omega : \text{dist}(x, \partial\Omega) < \delta\}$ , for  $\delta > 0$ . Suppose that  $M$  is a  $C^1$ -immersed minimal hypersurface with boundary in  $\mathbf{R}^{n+1}$ , and that  $M$  satisfies the following conditions:*

- (i)  $M \subset \Omega \times \mathbf{R}$ ,  $\partial M \subset \partial\Omega \times \mathbf{R}$ ;
- (ii) Near  $\partial M$ ,  $M$  is a graph over  $\Omega_\delta$ , for some  $\delta > 0$ .

*Then:*

- (a)  $M$  is a graph over  $\Omega$ . Hence  $M$  is embedded, and has the same topology as  $\Omega$ ;
- (b)  $M$  gives the unique solution to the following obstacle problem (see [L]):

$$(0.1) \quad \text{minimizing } \{\text{Mass}(T); T \in I_n(\mathbf{R}^{n+1}), \\ \partial T = \partial M \text{ and } \text{spt} T \subseteq \bar{\Omega} \times \mathbf{R}\}.$$

- (c)  $M = \text{graph}(U)$ , where  $U$  is the unique solution of the problem (see [LL]):

$$(0.2) \quad \text{minimizing } \int_{\Omega} \sqrt{1 + |dV|^2} dx + \int_{\partial\Omega} |V - g| dH^{n-1}$$

for  $V \in BV(\Omega)$ , where  $H^{n-1}$  denotes the  $(n-1)$ -dimensional Hausdorff measure, and  $\text{graph}(g) = \partial M$ .

The interesting point of the theorem above is that the limitation of the support and the boundary behavior of a minimal hypersurface leads to their geometrical and topological information.

Second, we wish to settle two questions which were posed by J. C. C. Nitsche [N, §887, §898].