

Projective Manifolds Containing Many Rational Curves

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Introduction. Varieties containing many rational curves come up repeatedly in algebraic geometry. In this paper we apply recent results on ample divisors to classify projective n -folds X containing many smooth rational curves with a given degree t with respect to an ample line bundle L . To quantify their number we consider the dimension p_t of the family of such curves of degree t passing through the general point of X . Having assumed $p_t \geq 0$, we show that $p_t \leq t(n+1) - 2$ and classify all the pairs (X, L) with

$$t(n-1) - 2 \leq p_t \leq t(n+1) - 2$$

(Theorem 1.4). These results provide a wide generalization of Sato's on manifolds containing many lines with respect to a very ample line bundle [8]. Further, though weaker, information is given about the structure of pairs (X, L) with

$$t(n-2) - 2 \leq p_t < t(n-1) - 2$$

(Proposition 2.1).

The main tool we use is a recent result by Ionescu [6] giving the structure of polarized n -folds (X, L) such that $K_X \otimes L^r$ ($n-2 \leq r \leq n+1$) fails to be semi-ample. Similar methods have been used recently in the literature, e.g., to investigate the structure of projective manifolds whose dual varieties have small dimension and hence contain large dimensional families of lines [4, 7]. In connection with this, our results allow us to recover the known classification of such manifolds up to dimension $n = 6$ and provide information for $n = 7$ and 8.

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§1. Let X be a complex projective n -fold ($n \geq 2$) polarized by an ample line bundle L ; for every positive integer t , $H_t(X, L)$ will denote the Hilbert scheme of the curves $C \subset X$, $C \simeq \mathbf{P}^1$ such that $C \cdot L = t$. Let $C_t(X, L)$ be the subscheme of the curves $C \in H_t(X, L)$ passing through a given general point $x \in X$ and put $p_t(X, L) = \dim C_t(X, L)$. If $X \subset \mathbf{P}^N$, we shall write for shortness $H_t(X)$ ($p_t(X)$) instead of $H_t(X, \mathcal{O}_X(1))$ ($p_t(X, \mathcal{O}_X(1))$), respectively. Finally, $G(k, n)$