

Complex Interpolation for Families of Quasi-Banach Spaces

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0. Introduction. In the last few years, R. Coifman, M. Cwikel, R. Rochberg, Y. Sagher and G. Weiss have developed a method of interpolation for families of Banach spaces that generalizes the complex method of A. P. Calderón (see [5] and [6]). As a continuation we present here a method of interpolation for families of quasi-Banach spaces. More precisely, we consider quasi-Banach spaces $B(\lambda)$ associated with the points λ of the boundary Γ of a suitable simply connected domain $D \subset \mathbb{C}$. Our aim is to construct “intermediate spaces” $B(z)$, for each $z \in D$, in such a way that some properties (especially boundedness) of an operator acting on the boundary spaces are also true when the operator acts on any one of the constructed spaces.

The extension to the quasi-Banach case is justified by the importance of some quasi-Banach spaces: L^p -spaces, weighted L^p -spaces, H^p -spaces, the Schatten ideals S^p for $p < 1$. In this paper we shall study in detail the interpolation of L^p -spaces. The interpolation of all the others will be studied in a second paper as the application of some results obtained there. We shall describe there a theory of interpolation for quasi-Banach lattices and a result on the connection between the real and the complex method of interpolation.

Interpolation of two quasi-Banach spaces, using the complex method, has been studied by M. Cwikel, M. Milman and Y. Sagher (see [7]). The Calderón method was extended to pairs of quasi-Banach spaces by N. Riviere in his thesis (see [12]). Unfortunately, his work was never published. His method used a non-homogeneous functional satisfying the triangle inequality (which gives the same topology as the quasi-norm). R. Rochberg (see [13]) has shown that the two methods lead to the same interpolation spaces.

The extension of the method developed in [5] and [6] to quasi-normed spaces is not routine for (at least) two reasons. Firstly, one cannot use duality as was done in the Banach space case. Secondly, we do not have, in the infinite dimensional case, a maximum principle (or a related vector-valued integration that gives a mean value inequality).

1. The finite dimensional case. Let B be a complex vector space on which is defined a map $|\cdot|$, with values in $[0, \infty)$, such that: