

Behavior of Functions in BMOA and VMOA near a $(2n-2)$ -dimensional Submanifold

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Let B_n denote the open unit ball in \mathbb{C}^n . In several recent papers ([A], [AR], and [R]), the behavior of functions in $\text{BMOA}(B_n)$ near various submanifolds of ∂B_n has been studied. In particular, in each of these papers, certain n (real)-dimensional submanifolds $M \subset \partial B_n$ and functions $f \in \text{BMOA}(B_n)$ have been produced with the property that f fails to have a finite radial limit at every point of M .

It seems natural to look for a submanifold of ∂B_n with the largest possible dimension for which the above can be done. In this paper we produce such an f and M , or at least come close to it—our techniques force us to replace “every” with “almost every”. More precisely, Theorem 1 below shows that for the submanifold $M = \frac{1}{\sqrt{2}}(\partial B_1 \times \partial B_{n-1})$, a $(2n-2)$ -dimensional submanifold of ∂B_n , there exists an $f \in \text{BMOA}(B_n)$ (and even in $\text{VMOA}(B_n)$) which fails to have a finite radial limit at H_{2n-2} -a.e point of M , where H_{2n-2} denotes $(2n-2)$ -dimensional Hausdorff measure on M . In fact, this f will have the property that it maps H_{2n-2} -a.e radius ending at a point of M onto a dense subset of \mathbb{C} .

As in [A], [AR], and [R], the desired $f \in \text{BMOA}$ is obtained after first proving a composition operator-theoretic result relating Bloch functions with functions in BMOA ; here we will be following the method of Ahern in [A]. However, unlike these earlier papers, where the authors relied on the existence of one variable functions displaying radial pathology, we here need to construct certain functions holomorphic in B_n , $n > 1$, exhibiting bad radial behavior almost everywhere on ∂B_n ; see Theorem 3.

We should point out that it is not just the existence of sets $M \subset \partial B_n$ of large Hausdorff dimension having the properties above which is of interest here. Indeed, it is known [S] that there are peak sets K for the ball algebra having Hausdorff dimension $2n-1$. For such a K there even exists an $f \in H^\infty(B)$ which fails to have radial limits (or limits along any curve) at every point of K . But there exists no such $f \in H^\infty(B_n)$ with this property relative to our submanifold