

# *Factorization over Algebras of Subnormal Operators*

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**1. Introduction.** Let  $\mathcal{B}(\mathcal{H})$  denote the algebra of bounded linear operators on a separable, infinite-dimensional complex Hilbert space  $\mathcal{H}$ . Regarding  $\mathcal{B}(\mathcal{H})$  as the dual of the space of trace class operators, we can give  $\mathcal{B}(\mathcal{H})$  a weak-star topology. For  $S$  in  $\mathcal{B}(\mathcal{H})$  let  $\mathcal{A}(S)$  denote the weak-star closed algebra generated by  $S$  and  $I$ . The purpose of this paper is to give a new proof of the following factorization theorem of Olin and Thomson [7].

**Theorem 1.** *Let  $S$  be a subnormal operator in  $\mathcal{B}(\mathcal{H})$  and let  $L$  be a weak-star continuous linear functional on  $\mathcal{A}(S)$ . Then there exist vectors  $x$  and  $y$  in  $\mathcal{H}$  with  $\|x\|$  and  $\|y\|$  each bounded by a universal constant times  $\|L\|^{1/2}$  such that*

$$L(T) = \langle Tx, y \rangle$$

*for each operator  $T$  in  $\mathcal{A}(S)$ .*

The new proof arises from the factorization method in [9]. The original proof of this theorem led to the existence of analytic invariant subspaces for subnormal operators and then to the theorem that every subnormal operator is reflexive. Extending our new factorization method in similar ways would yield those results as well. A description of the connection between factorization and reflexivity can also be found in [10].

Recently, S. Brown, B. Chevreau, and C. Pearcy have proved the striking theorem that every contraction whose spectrum contains the unit circle has non-trivial invariant subspaces [1], [2]. In their present work and also in the past work of their group, they have obtained related factorization results.

The book by J. B. Conway [4] is an excellent reference for background material for this paper.

**2. Preliminaries.** Let  $S$  be a subnormal operator in  $\mathcal{B}(\mathcal{H})$  with minimal normal extension  $N$ , and let  $\mu$  denote the scalar spectral measure for  $N$ . If  $P^\infty(\mu)$  denotes the weak-star closure of the polynomials in  $L^\infty(\mu)$ , then

$$\mathcal{A}(N) = \{\varphi(N) : \varphi \in P^\infty(\mu)\}$$