

Fractional Powers of $(-\Delta)^m + M_q^{2m}$ with Applications to Trace Class

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0. Introduction. In our recent paper (Takáč [22, Theorem 1.3]) we derived a simple formula for the resolvent $(H + \lambda I)^{-1}$, $\lambda > 0$, of the selfadjoint elliptic operator $H = (-\Delta)^m + M_q^{2m}$ on $L_2(\mathbf{R}^N)$, where $N \geq 1$ and $m \geq 1$ are integers, Δ is the Laplacian, and M_q denotes the operator of pointwise multiplication by a positive continuous function q on \mathbf{R}^n . The goal of the present paper is to generalize the main results of the previous one to fractional powers of $H + \lambda I$ and to weighted Besov (or Sobolev–Slobodeckij) spaces. We state a formula for the operator $(H + \lambda I)^{-\omega}$, $0 < \omega \leq 1$, as a part of our Theorem 1.3. It conveys sufficient information about the kernel of the integral operator $(H + \lambda I)^{-\omega}$, so that we can prove a Hilbert–Schmidt criterion for the inverse $A_{\lambda, \omega}^{-1}$ of the (nonselfadjoint) closed operator

$$A_{\lambda, \omega} = (H + \lambda I)^\omega M_p$$

on $L_2(\mathbf{R}^N)$. Here p is another positive continuous function on \mathbf{R}^N . We state this criterion as Theorem 1.4. To complete our study of the operator $A_{\lambda, \omega}$ we identify its domain in Theorem 1.5. As an application of Theorems 1.4 and 1.5 we formulate a trace class criterion for an integral operator K on $L_2(\mathbf{R}^N)$ which we state as Theorem 1.6. In this criterion we formulate sufficient conditions on the kernel $k(x, y)$, $x, y \in \mathbf{R}^N$, of the integral operator K which imply that K is of trace class. These conditions require that the kernel $k(x, y)$ have both sufficient fractional smoothness and decay at infinity with respect to the x -variable. As a direct consequence of our Theorem 1.6, we state Corollaries 1.7 and 1.8. The latter one shows overlapping between our results and those of Kamp, Lorentz and Rejto [8]. Finally we illustrate the optimality of our trace class criterion with Example 1.9.

As for the organization and methods of this paper, we state our main results as Theorems 1.3 through 1.6 in Section 1. We refer to Takáč [22] for the proofs of Proposition 1.2 and Theorem 1.3 (the formula for $(H + \lambda I)^{-1}$).

In Section 2 we prove Theorem 1.4. To find a sufficient condition for $\alpha = 2m\omega$, p and q that the operator $A_{\lambda, \omega}^{-1} = M_p^{-1}(H + \lambda I)^{-\omega}$ on $L_2(\mathbf{R}^N)$ be Hilbert–