

# *Rates of Asymptotic Convergence Near Isolated Singularities of Geometric Extrema*

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**Introduction.** In [SL1] (see also [SL2]) there are results concerning the asymptotic behavior of minimal surfaces and harmonic maps (and extrema of other geometric variational problems) near isolated singular points. General conditions were established which ensured that these extrema would converge to their tangent cones and tangent maps on approach to an isolated singular point. However, the method used did not yield much information about the rate at which this convergence occurred, and this led to the speculation (see the introduction of [SL1]) that the convergence might be slow, even like a power of  $1/|\log|x||$  as  $|x| \rightarrow 0$ . Earlier it was shown in [AA] that for minimal surfaces the convergence rate was at least  $|x|^\alpha$  for some  $\alpha > 0$ , provided a certain “integrability condition” held. This work was extended to other functionals and simplified in [SL2, Part 2, Section 5], working with the natural generalization of the integrability condition, as in  $(\star)$  of Section 1 below.

Conveniently enough, it turns out that this integrability condition is essentially the *necessary* and *sufficient* condition to guarantee convergence like  $|x|^\alpha$ : this is the content of Theorem 1. (In reading this theorem the reader should keep in mind that, in applying Theorem 1 to minimal surfaces and harmonic maps, one uses the variable  $t = \log 1/|x|$ , so convergence like  $|x|^\alpha$  means *exponential* convergence in  $t$ :  $|x|^\alpha = e^{-\alpha t}$ .)

Theorem 2 of the present paper gives more precise information about the class of “slow decay” solutions (i.e., solutions which decay like a negative power of  $t$  rather than like  $e^{-\alpha t}$ ) in case the integrability condition fails. It is proved that there is a very large class of such slow decay solutions in this case (modulo a possible transformation  $t \mapsto -t$  in certain cases). Specifically, if the integrability condition fails, Theorem 2 shows that there are solutions having slow decay and having boundary values differing from a given function  $g$  of small  $C^{2,\alpha}$  norm by elements in a fixed finite-dimensional subspace of  $L^2$  (the subspace being independent of  $g$ ).