

Partial Hölder Continuity for Minima of Certain Energies among Maps into a Riemannian Manifold

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The present paper gives an interior partial regularity result for minimizers of certain functionals between Riemannian manifolds. The result is known [10] for the Dirichlet integral. But also in this case, I think, the approach presented here constitutes a simplification of the proof. The case of the L_p norm of the gradient has been treated independently by Hardt and Lin [6]. Their proof is closer to the original one in [10] for the Dirichlet integral.

The method of proof here is also to use blow up sequences around points z in the preimage manifold M . Two cases are distinguished. The first is that the normalized p -Dirichlet integral,

$$\rho^{p-n} \int_{B_\rho(z)} |\nabla u|^p, \quad 1 < p < \infty,$$

converges to 0. In that case, following Hardt, Kinderlehrer and Lin [5]—see also [4] in the euclidean case—one rescales as well in the image as in the domain, to get a sequence of functions with unit norm in $H_p^1(B_1)$. This has the effect to flatten the target manifold N . And one is able to infer Morrey's Dirichlet growth estimate for u from the same estimate for solutions to the limit equation in euclidean space. The result then is that the $n - p$ Hausdorff measure of the set of singular points is zero.

To reduce the Hausdorff dimension further, following Schoen and Uhlenbeck [10], one also considers blow up sequences in points where the p -Dirichlet integral does not tend to zero. In this case one rescales only in the domain. And the limit is an energy minimizer, which is radially homogeneous of order zero. That reduces the dimension of the singular set to $n - [p] - 1$ by Federer's dimension reduction argument [2].

In both cases the difficulty is to show strong convergence of the blow up sequences in H_p^1 and that the limit minimizes the energy. That is done by an energy comparison argument. One has to construct maps into N whose energy is close to that of the limit map, but whose boundary values are those of the approximations.