

# *Contractive Metrics for Nonlinear Hyperbolic Systems*

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**1. Introduction.** Consider the Cauchy problem for a strictly hyperbolic  $n \times n$  system of conservation laws:

$$(1.1) \quad u_t + (F(u))_x = 0,$$

$$(1.2) \quad u(x, 0) = u_0(x).$$

Assuming that (1.1) is genuinely nonlinear (cf. [5]) and that the initial condition  $u_0$  has suitably small total variation, a well-known theorem of Glimm [3], [11, p. 368] provides the existence of a weak solution of (1.1), (1.2), defined for all  $t \geq 0$ . This result is proved by introducing a nonlinear functional, which measures the amount of wave interaction, and by establishing an *a priori* bound on the total variation of the solution. A continuous version of the interaction potential was recently studied in [10]. In this paper we discuss the possibility of using continuous Glimm's functionals in order to construct a metric, equivalent to the distance in  $\mathcal{L}^1$ , which is nonexpansive with respect to the evolution system (1.1). This will shed some new light on the question of uniqueness and continuous dependence for weak solutions of (1.1). Most of the analysis will be concerned with classical solutions. These are precisely the  $C^1$  solutions of the quasilinear system

$$(1.3) \quad u_t + A(u)u_x = 0,$$

where  $A(u) = DF(u)$  denotes the  $n \times n$  Jacobian matrix of  $F$  at the point  $u \in \mathbf{R}^n$ . If  $u = u(x, t)$  is such a solution, together with (1.3) we consider the linear, time-dependent system

$$(1.4) \quad v_t(x, t) + [DA(u(x, t)) \cdot v(x, t)]u_x(x, t) + A(u(x, t))v_x(x, t) = 0$$

obtained by linearizing (1.3) about the given trajectory  $u$ . In the following,  $C_0^2$