

# *Some Exotic Dihedral Actions on Spheres*

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A *representation form* is an action of a group  $G$  on a sphere  $X$  such that the fixed point set  $X^H$  of each subgroup  $H \subset G$  is again a sphere. A guiding question is the extent to which general forms resemble those given by linear actions. In this note we study representation forms of  $D_{2m}$ , the dihedral group of order  $2m$ .

The first major result was due to J. Milnor [1957]: the dihedral group cannot act freely on the sphere. Thus a natural question arises: if the dihedral group acts on a sphere, what can be said about the fixed point sets of the various subgroups?

Natural linear actions of  $D_{2m}$  on  $S^{2n-1}$  are given by considering  $D_{2m}$  as a subgroup of  $O(2)$  as symmetries of the regular  $m$ -gon, and then embedding  $D_{2m}$  diagonally in  $O(2) \times \cdots \times O(2) \subset O(2n)$ . D. Montgomery–C.T. Yang [1981] showed that general dihedral actions resemble linear actions in the following sense:

**Theorem.** *Let  $X$  be a smooth representation form of  $D_{2m}$  ( $m$  odd,  $m > 1$ ) of dimension  $2n - 1$  so that  $X^H = \emptyset$ , where  $H$  is the cyclic subgroup of order  $m$ . Then for any two elements  $g, h \in D_{2m}$  of order 2,  $\dim X^g = n - 1 = \dim X^h$  and the linking number  $\ell(X^g, X^h)$  is odd.*

Of course in the linear examples  $X^g$  and  $X^h$  are disjoint linearly embedded subspheres so the linking number is 1. A natural question posed by S. Cappell (see Schultz [1985], p. 590) is whether there exist  $D_{2m}$ -actions with exotic linking numbers.

We now outline the construction of exotic actions. The starting point of this investigation is to consider the symmetry properties of Brieskorn varieties.