

Elliptic Perturbations of Nonlinear Oscillations in the Presence of Resonances

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1. Introduction. Let M be a C^∞ manifold of dimension k and dL a smooth volume on M . Consider the manifold $\hat{M} = M \otimes \mathbf{T}^n$, (\mathbf{T}^n -the n -dimensional torus). Letting x, y be the coordinates of M, \mathbf{T}^n respectively, define the volume element $d\hat{L}$ on \hat{M} as:

$$d\hat{L} = dL \wedge dy_1 \cdots \wedge dy_n.$$

Let ∂_H be a vector field on \hat{M} :

$$(1.1) \quad \partial_H \equiv \sum_1^n \Omega_i(x) \frac{\partial}{\partial y_i}.$$

Assume $\{\Omega_i\}_{i=1,n} \in C^\infty(M)$ and for every vector of integers $\vec{\ell} = (\ell_1, \dots, \ell_n)$:

$$(1.2) \quad \vec{\ell} \cdot \Omega(x) := \sum_1^n \ell_i \Omega_i(x) \neq 0 \text{ almost surely } (dL) \text{ on } M.$$

Notice that, if $k \geq n$, (1.2) is satisfied provided

$$(1.2') \quad \text{Rank} \left\{ \frac{\partial \Omega_i}{\partial x_j} \right\} (x) = n \text{ almost surely } (dL) \text{ on } M.$$

$x \in M$ is called a resonant point if there exists an integer vector $\vec{\ell}$ for which $\vec{\ell} \cdot \Omega(x) = 0$.

Let L^1 be an elliptic operator on \hat{M} :

$$L^1 = Q^1(x, y) \nabla_x + Q^2(x, y) \nabla_y + \nabla_z a \cdot \nabla_z$$

where $z = (x_1 \dots x_n, y_1 \dots y_n)$, $\nabla_z = (\nabla_x, \nabla_y)$,

$$\nabla_z a \cdot \nabla_z = \sum_{i,j=1}^{n+k} \frac{\partial}{\partial z_i} a_{ij} \frac{\partial}{\partial z_j}.$$