

# *Area-minimizing Cone-type Surfaces and Coflat Calibrations*

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**0. Introduction.** Let  $M$  be a submanifold of the unit sphere  $S^{n-1} \subseteq \mathbb{R}^n$ . Then the cone over  $M$  is defined by

$$0 \star M = \{t: x \in M, t \geq 0\}.$$

In this paper, we prove the existence of two single and seven infinite families of new area-minimizing equivariant conical varieties in Euclidean spaces, which are smooth everywhere except for an isolated singularity at the origin. They are cones over submanifolds of type:

- (1)  $SU(n)$  in  $\mathbb{R}^{2n^2}$ ;
- (2)  $SU(n)/SO(n)$  in  $\mathbb{R}^{n^2+n}$ ;
- (3)  $U(n)$  in  $\mathbb{R}^{2n^2}$  for  $n \geq 4$ ;
- (4)  $U(n)/O(n)$  in  $\mathbb{R}^{n^2+n}$  for  $n \geq 7$ ;
- (5)  $U(7)/U(1) \times SU(2)^3$  in  $\mathbb{R}^{42}$ ;
- (6)  $Sp(n) \times Sp(3)/Sp(1)^3 \times Sp(n-3)$  in  $\mathbb{R}^{12n}$  for  $n \geq 4$ ;
- (7)  $Sp(4)/Sp(1)^4$  in  $\mathbb{R}^{27}$ ;
- (8) products of spheres  $S^p \times$  any number of copies of  $S^q$  for  $p \geq 4$  and  $q \geq 4$ ;  
and
- (9) a class of twisted normal cones in  $\mathbb{R}^{4n}$ .

All nine are proved to be area-minimizing using the techniques of the theory of calibrations. Cones (1), (2), and (7) are special Lagrangian varieties. The others are proved by constructing new types of coflat (or singular) calibrations in Euclidean space. Note cones (2), (4), (7), and (8) extends the list of all known area-minimizing cones given in [Ch].

Surfaces which are smooth everywhere except for isolated singularities are special cases of more general objects called rectifiable currents, first introduced by H. Federer and W. Fleming [F-Fm]. By an area-minimizing surface, we mean any compact portion of the surface minimizes area in comparison with any rectifiable current having the same boundary. These cones also provide new examples of the non-interior regularity of solutions to Plateau's problem.