

A Kinetic Model for the Compressible Flow Equations

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1. The equations governing the motion of a smooth inviscid compressible flow are

$$(1.1) \quad \frac{\partial \rho}{\partial t} + \frac{\partial m_i}{\partial x_i} = 0$$

$$(1.2) \quad \frac{\partial m_i}{\partial t} + \frac{\partial}{\partial x_i} \left(\frac{m_i m_k}{\rho} \right) + \frac{\partial p}{\partial x_k} = 0$$

$$(1.3) \quad \frac{\partial S}{\partial t} + u_i \frac{\partial S}{\partial x_i} = 0$$

$$(1.4) \quad p = F(p, S).$$

The scalar $\rho(x, t)$ denotes the density; the vector $m(x, t)$ denotes the momentum; the vector $u(x, t)$ denotes the velocity $u = \frac{m}{\rho}$; $p(x, t)$ is the pressure; $S(x, t)$ is the entropy. F is a general equation of state. x is a point in n -dimensional space $n = 1, 2, 3$.

A kinetic model for fluid dynamics was first constructed by Maxwell [4]. He assumed the density $\rho(x, t)$ and the momentum $m(x, t)$ to be the moments of a distribution function $f(x, \xi, t)$:

$$(1.5a) \quad \rho(x, t) = \int f(x, \xi, t) d\xi$$

$$(1.5b) \quad m(x, t) = \int \xi f(x, \xi, t) d\xi.$$

Maxwell assumed $f(x, \xi, t)$ to evolve by, what was later known to be, the Boltzmann equation. He, however, thought that $f(x, \xi, t)$ and, consequently, the Boltzmann equation do not have direct physical meaning. He therefore produced the equations of transfer which are the zero and first moments of the Boltzmann equation. He then assumed $f(x, \xi, t)$ to be in a normal form, which later was called "a Maxwellian". This normal form was used for the computation of the coefficient of viscosity and the coefficient of the heat transfer.