

Homomorphisms and Semigroups in Weighted Convolution Algebras

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1. Introduction. In this paper we study continuous homomorphisms between spaces of locally integrable functions on the half line $R^+ = [0, \infty)$ which are algebras under the convolution product

$$f * g(x) = \int_0^x f(x-t)g(t) dt.$$

We are particularly interested in relations between the structure of these algebras and the properties of the homomorphisms. In [2, Section 4], Bade and Dales apply their results on norms of elements to characterize when such homomorphisms exist, and, in a number of papers, [7], [8], [9], Ghahramani studies the isomorphisms between such algebras. A number of other recent papers have studied the structure of weighted convolution algebras with particular emphasis on the description of the closed ideals and subalgebras ([1], [3], [4], [5], [11], [13]). A major motivation for the study of isomorphisms and homomorphisms between weighted convolution algebras [12, Question 1, p. 288] has been the hope that they will prove as useful in the study of weighted convolution algebras as they did in the study of their discrete analogues, Banach algebras of power series [10]. Some of the results in this paper show an intimate connection between the nature of homomorphisms between radical convolution algebras and the nature of ideals in these algebras.

Suppose that $\omega(x)$ is a positive Borel function which is essentially bounded, and essentially bounded away from 0, on compact subsets of $(0, \infty)$. Then $L^1(\omega)$ is the Banach space of (equivalence classes of) all complex measurable functions f for which $f\omega$ belongs to $L^1(R^+)$, with the inherited norm

$$\|f\| = \|f\|_\omega = \int_0^\infty |f(t)|\omega(t) dt.$$