

# *Some Remarks about Harmonic Maps into Flag Manifolds*

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**1. Introduction.** This note is concerned with the problems of understanding harmonic maps from compact Riemann surfaces into flag manifolds. Since minimal surfaces are harmonic in some conformal structure, special cases are provided by minimal surfaces.

In the late 1960's Chern [7] and Calabi [5] published several papers on minimal immersions into spheres or more generally real projective spaces, which are in the spirit of this investigation. Then Simons [18], Lawson [17], Hsiang and Lawson [16] published important papers in this direction.

The problem was reexamined by physicists Glaser and Stora [13] and Din-Zakarewski [9]. They called the attention that the right problem should be to look at harmonic maps into complex projective spaces. Inspired by these ideas Eells and Wood [11] gave a complete classification for harmonic maps from  $CP^1$  to  $CP^n$ , and some important partial results for the higher genus cases in terms of holomorphic data. A number of related results have appeared including Burstall and Wood [4], Chern and Wolfson [8], Uhlenbeck [21]. These authors have studied harmonic maps into other homogeneous symmetric spaces like Lie groups and Grassmannians.

On the other hand, very little is known about the homogeneous (non-symmetric) case. One reason that we want to understand the non-symmetric case, beside its own intrinsic interest, is that the finite dimensional flags model in finite dimensions the geometry of the Loop group; i.e., maps from  $S^1$  to a compact Lie Group  $G$ . In a well-known paper Atiyah calls attention to the identification of holomorphic maps into the loop group and instantons [2]. See also Freed's paper [12] for more details.

Secondly the energy functional whose critical points are the harmonic maps is more tractable from the point of view of computations than for example the Yang-Mills functional, but appears to share some of their important properties. See Wolfson's paper [22] for more details.

If one wants to understand the problem of harmonic maps into a homogeneous (non-symmetric) space it is natural to start by understanding harmonic maps from closed Riemann surfaces to flag manifolds. These are by definition