

# On Normal Gorenstein Polarized Varieties of Sectional Genus 3 and 4

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**Introduction.** Throughout the paper we deal with a complex, normal, Gorenstein variety  $X$  of dimension  $n$  with at most a finite number of non-rational singularities, endowed with a very ample line bundle  $L$ . Let  $g(L)$  be the sectional genus of  $L$ . Our main interest is to classify such polarized pairs  $(X, L)$  when  $n \geq 3$  and  $g(L) = 3$  or  $4$ . We use heavily the methods and the analogous results for surfaces contained in [S4]. The main tool is to study the adjunction mapping of  $(X, L)$  (see [S1] and [S2]). It should be noted that relaxing either the Gorenstein condition or the condition on the non-rational locus leads to examples where any reasonably defined adjunction mapping is meromorphic. Indeed in greater generality precise results such as we obtain are not true (see [A-S]).

The cases  $g(L) \leq 2$  are essentially well known in various levels of generality (see [F1], [F2], [Be-La-P], [I1], [Be-S], [H]). In the smooth case a great deal is known for dimension 2 and  $g(L) \leq 6$ , ([I2], [E], [Li1], [Li2]) and essentially as corollaries of these results for  $n$ -folds ([I2], [I3]).

Looking over the proofs we give, one has the following for  $g(L) = 2$ .

**Theorem.** *Let  $(X, L)$  be as in (0.3) with  $g(L) = 2$ . Then either:*

- (i)  $(X, L)$  is a scroll over a nonsingular curve of genus 2, or
- (ii)  $(X, L)$  is a quadric bundle over  $\mathbf{P}^1$ .

Rather than to quote here the results we get for  $g(L) = 3$  and  $4$  we suggest the reader to look at Theorems (2.1) and (3.1).

The paper is organized as follows. In Section 0 we recall some background material. In Section 1 we give some general results on the  $(n-1)$ -pluridegrees  $\delta_j = (K_X + (n-1)L)^j \cdot L^{n-j}$ ,  $j = 0, \dots, n$ , of a polarized pair  $(X, L)$ . The main result is concerned with the more difficult case when  $K_X + (n-1)L$  is nef and big. Under the extra assumption that the singular locus of  $X$  does not contain any linear  $\mathbf{P}^{n-2}$  and up to a small list of very understood varieties the inequality  $\delta_j \geq \delta_{j-1}$ ,  $j \geq 1$ , can be proved. The inequality above becomes as useful as  $g(L)$  increases and allows us to rule out a number of cases when  $g(L) = 4$  or  $5$  and  $\text{cod Sing}(X) \geq 3$ . Sections 2 and 3 are devoted to cases  $g(L) = 3$  and