

Minimal Hypercones and $C^{0,1/2}$ Minimizers for a Singular Variational Problem

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1. Introduction. It is well known that the one-dimensional problem

$$\mathcal{E}_1(u) = \int_a^b u(t) \sqrt{dt^2 + du^2} \rightarrow \text{minimum}$$

leads to rotationally symmetric minimal surfaces bounded by two coaxial circles. Solutions are provided by revolving the catenaries

$$u = a \cdot \cosh \left(\frac{t - t_0}{a} \right)$$

about the t -axis.

Introducing a Lagrangian multiplier the problem may also be formulated as follows. What are the curves of prescribed length ℓ , connecting two given points P_1, P_2 and having lowest center of gravity? Hence the catenaries also have stationary potential energy under gravitational forces and prescribed isoperimetric conditions.

However, Goldschmidt, in a celebrated paper [Go], had discovered the existence of the so-called discontinuous solution, which consists of three straight lines L_1, L_2, L_3 , namely the shortest connections L_1, L_2 between P_1, P_2 and the t -axis respectively and the corresponding part of the t -axis itself. It turns out that the discontinuous solution is a relative minimum, which is even an absolute one provided the horizontal distance of the points P_1 and P_2 is large, see the book of Bolza [Bo] for a more careful discussion (cf. pp. 436–438).

The n -dimensional analogue has only recently been solved by Bemelmans and Dierkes [BD] and I refer to that paper for further information on the literature. In [BD] the authors prove the existence of a continuous minimizer for the potential energy functional

$$\int_{\Omega} u \sqrt{1 + |Du|^2} + \frac{1}{2} \int_{\partial\Omega} |u^2 - \psi^2| d\mathcal{H}_{n-1}$$