

A Class of Weakened Special Lagrangian Calibrations

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I. INTRODUCTION

In the recent papers [7] and [10], the method of calibrations has made one of its first significant contributions to the theory of singularities of area-minimizing surfaces, namely a complete solution to the following problem: which pairs of (oriented) k -dimensional planes in \mathbf{R}^n are area minimizing? As proven in these papers, a necessary and sufficient condition for area minimization is that k “characterizing angles” of intersection of the two planes, when placed in increasing order, satisfy a single linear inequality, now known as the Angle Criterion.

The proof of the sufficiency of the Angle Criterion in [10] involved the construction of a new class of constant-coefficient calibrations in $\Lambda^n \mathbf{R}^{2n}$, which were there called “ Q -forms” (see Definition 2.2). These may be considered to be “weakened” (or “modified”) *Special Lagrangian* calibrations in two senses. They are obtained by replacing the imaginary unit i in the conventional expression for the Special Lagrangian form by arbitrary quaternions u_1, \dots, u_n ; also, they calibrate pairs of planes which are in the interior of the space of area-minimizing pairs, while the Special Lagrangian form calibrates pairs which are on the boundary. The purpose of the present paper is to investigate further the properties of Q -forms and their generalizations. The questions we study fall under three main headings:

(1) *Dimensions of faces of torus forms.* The main result is Theorem 3.8:

If $\varphi \in \Lambda^4 \mathbf{R}^8$ is a torus form and the number of points in the face G_φ is infinite, then φ is a Q -form, an “ L -form” or one of the forms catalogued in [3, Chap. 5] (an “ S^1 -form”).

Theorem 3.13 extends this result to torus forms in $\Lambda^n \mathbf{R}^{2n}$ such that $\dim(G_\varphi) \geq n - 3$.