

Mapping Properties of the Szegő Operator for the Bidisc

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ABSTRACT. Consider the Szegő operator for the topological boundary of the bidisc in \mathbf{C}^2 with respect to Lebesgue surface area measure. It is shown that this operator is bounded on L^p and the Sobolev spaces L_k^p for $1 < p < \infty$ and $k \in \mathbf{Z}^+$. Furthermore, the operator maps $L(\log^+ L)^2$ into L^1 . Counterexamples show that the operator is not weak-type (1,1) and is not bounded on the Lipschitz spaces Λ_γ for any $0 < \gamma < 1$.

1. Introduction. Let Ω be a bounded sufficiently smooth domain in \mathbf{C}^2 and let $\partial\Omega$ be its boundary. The space $H^p(\partial\Omega)$ is defined to be the set of all functions f such that f is in $L^2(\partial\Omega)$ and f is the extension of a holomorphic function on Ω . The Szegő operator for $\partial\Omega$ is defined to be the orthogonal projection of $L^2(\partial\Omega)$ into $H^2(\partial\Omega)$.

Korányi and Vági [1] have shown L^p boundedness of the Szegő operator for the ball and generalized half-plane. Phong and Stein [3] have shown for any strongly pseudo-convex domain $\Omega \subseteq \mathbf{C}^n$, $n \geq 2$, that the Szegő operator is a bounded operator on $L^p(\partial\Omega)$ for $1 < p < \infty$, more generally on Sobolev spaces $L_k^p(\partial\Omega)$, and on Lipschitz spaces $\Lambda_\gamma(\partial\Omega)$.

In this paper, we consider the topological boundary of the bidisc with Lebesgue surface area measure. We first show that the Szegő operator \mathcal{S} for this boundary maps L^p into L^p for $1 < p < \infty$. It should be pointed out that this problem is very different from the L^p mapping properties of the Szegő operator for the distinguished boundary of the bidisc. It is well known (see [2], p. 55) that the Szegő operator may be expressed as an integral operator of the form

$$\mathcal{S}f(z) = \int_{\partial\Omega} \tilde{S}(z, \zeta) f(\zeta) d\sigma(\zeta),$$

where \tilde{S} is the Szegő kernel. To prove the result we find an integral representation for the Szegő kernel \tilde{S} . This kernel may be expressed as a sum of several terms. Each of the main terms of the kernel is a product of two kernels. The first kernel is a Calderón-Zygmund kernel in one of the angular variables. The second kernel