

Isometric Asymptotes of Power Bounded Operators

LÁSZLÓ KÉRCZY

ABSTRACT. It is a frequently used, fruitful idea in operator theory that studying general operators, we relate them to special ones and then exploit this connection. In the present paper isometries are invoked to investigate power bounded operators. These isometries will be attached to operators in an asymptotic way and do not degenerate if the powers of the operator considered do not converge to zero in the strong operator topology. After giving the basic definitions and theorems in the first section, in Section 2 we describe how the spectra of power bounded operators relate to the spectra of their asymptotes, while in Section 3 the connection between the hyperinvariant subspace lattices is treated.

I. Auxiliary Definitions and Theorems

1. Let H be a complex Hilbert space endowed with the inner product $\langle \cdot, \cdot \rangle$, and let $B(H)$ denote the algebra of all bounded linear operators acting on H . We recall that an operator $T \in B(H)$ is called *power bounded* if the powers of T form a bounded sequence $\{T^n\}_{n=0}^\infty$ in the Banach space $B(H)$. The idea of introducing a new inner product on H by using a Banach limit stems from a paper of B. Sz.-Nagy. He applied this method in [17] proving that every power bounded operator with a power bounded inverse is similar to a unitary operator. (See also [2] for a more recent application in the case of contractions.)

Let L denote a Banach limit, that is, a positive linear functional on the sequence space $\ell^\infty(\mathbf{N})$ with the properties $L(1, 1, \dots) = 1$ and $L(c_1, c_2, c_3, \dots) = L(c_2, c_3, \dots)$. (For details see [1, p. 33] and [6, Section III.7].) Let us consider an arbitrary power bounded operator T on H , $\sup_{n \geq 0} \|T^n\| < \infty$. Then defining

$$[x, y] := L(\{\langle T^n x, T^n y \rangle\}_n) \quad (x, y \in H),$$

we get a new semi-inner product on H . Thus the factor space H/H_0 , where H_0