

# *Properties and Examples of ( $L^p, L^q$ ) Multipliers*

KATHRYN E. HARE

ABSTRACT. We discuss conditions pertaining to the rate of decay of bounded sequences which almost characterize the space of multipliers from  $L^2(G)$  to  $L^q(G)$ , denoted by  $M(2, q)$ , for  $G$  a compact abelian group and  $q > 2$ . Specific examples of multipliers are constructed to illustrate that  $M(2, q) \subsetneq M(2, r)$  if  $q > r$ . The spectrum of a multiplier  $\varphi \in M(2, p)$ , for  $p > 2$ , as an operator from  $L^p$  to  $L^p$  is seen to be  $\{\varphi(\widehat{G})\}^{cl}$ . It is shown that for some values of  $p$  and  $q$  there are examples of multipliers from  $L^p$  to certain quotient spaces of  $L^q$  which are not the restrictions of multipliers from  $L^p$  to  $L^q$ . Quasi-idempotent multipliers from  $H^1(T)$  to  $H^p(T)$  for  $p > 1$  are characterized.

**0. Introduction.** Let  $G$  be a compact abelian group and let  $\Gamma$  be its discrete dual group. Let  $1 \leq p, q \leq \infty$ . A function  $\varphi : \Gamma \rightarrow \mathbf{C}$  is called an  $(L^p, L^q)$  multiplier if for every  $f \in L^p(G)$ ,  $M_\varphi f \in L^q(G)$  where by  $M_\varphi f$  we mean the function whose Fourier transform satisfies

$$(M_\varphi f)^\wedge(\chi) = \varphi(\chi)\hat{f}(\chi)$$

for every  $\chi \in \Gamma$ . The space of  $(L^p, L^q)$  multipliers will be denoted by  $M(p, q)$ . It is well known that  $M(p, q) = M(q', p')$  if  $\frac{1}{p} + \frac{1}{p'} = 1$  and  $\frac{1}{q} + \frac{1}{q'} = 1$ . Also,  $M(2, 2) = \ell^\infty \subsetneq M(p, p)$  if  $p \neq 2$ ,  $M(1, 1) = M(G)$  and  $M(1, p) = L^p(G)$  if  $p > 1$ . For proofs of these results see [13] and [3]. For other choices of  $p$  and  $q$  effective characterizations of  $M(p, q)$  are unknown.

The main purpose of this paper is to describe and apply new conditions, some necessary, others sufficient, which almost characterize the  $(L^p, L^q)$  multipliers. From these we obtain, for example, a characterization of quasi-idempotent multipliers. The spectrum of  $\varphi \in M(2, p)$ ,  $p > 2$ , viewed as an operator from