

Potential Theory for Supersolutions of Degenerate Elliptic Equations

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1. Introduction. Let $A : \mathbf{R}^n \times \mathbf{R}^n \rightarrow \mathbf{R}^n$ be an operator with degenerated ellipticity, $A(x, h) \cdot h \approx |h|^p$, $1 < p \leq n$; see Section 2 for the precise assumptions. Weak solutions of the (usually nonlinear) equation

$$(1.1) \quad \nabla \cdot A(x, \nabla u) = 0$$

are always continuous and we call them A -harmonic [S], [HK1]. A typical example of the equations considered is the p -Laplace equation

$$\nabla \cdot (|\nabla u|^{p-2} \nabla u) = 0;$$

the solutions of which are customarily called p -harmonic.

A function u in an open set $G \subset \mathbf{R}^n$ is A -superharmonic if

$$(1.2) \quad u \text{ is lower semicontinuous (lsc),}$$

$$(1.3) \quad -\infty < u \leq +\infty, \text{ and}$$

$$(1.4) \quad \text{for each domain } D \Subset G \text{ and for each } A\text{-harmonic function } h \in C(\bar{D}), \\ h \leq u \text{ in } \partial D \text{ implies } h \leq u \text{ in } D.$$

In a joint paper with J. Heinonen [HK1] we have shown that weak supersolutions of (1.1) are A -superharmonic when properly pointwise redefined, see Proposition 2.9 below, and therefore the definition above allows us to study also supersolutions as pointwise defined functions. In a nonlinear situation this approach was apparently first used when S. Granlund, P. Lindqvist and O. Martio [GLM1] introduced subextremals for variational integrals of the type $\int F(x, \nabla u) dx \approx \int |\nabla u|^n dx$. In subsequent papers they considered potential theoretical aspects of subextremals with applications to function theory [GLM1–2], [LM1–2], [Lq]. Later J. Heinonen and the present author took up the study of general A -superharmonic functions and their nonlinear potential theory; several analogues to classical potential theory were found, see [HK1–3]. This approach