

The Hopf Algebra Structure of the Complex Bordism of the Loop Spaces of the Special Orthogonal Groups

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ABSTRACT. The Hopf algebra structure of $MU_*(\Omega SO(n))$ is computed, generalizing the results of Bott for ordinary homology and of Clarke for K -theory. In fact, rephrased in terms of formal group structure, their results hold for any MU -algebra theory.

1. Introduction. There has been a steady interest in calculating the BP -homology of well known spaces. Most of the effort has been directed towards spaces with torsion. However, determining *all* the relevant structure in *easily* expressible terms is not straight forward even for torsion-free spaces, as the cases of loop spaces of orthogonal and symplectic groups shows. The latter has been studied in [Cl2]. Here we will study the former. Actually, I have not been able to determine the module of coalgebra primitives, but the rest of the structure is determined in terms of the formal group structure associated to complex cobordism. These results then transfer to any MU -algebra theory, including ordinary homology and K -theory. These two cases have been settled by [Bt] and [Cl1] respectively, and their results are special cases of ours. Actually, we do use Bott's calculations; but with a little more effort we can do with only the general results of Bott applicable to any Lie group and the rational calculations of Leray, Borel et al.

The plan of this paper is as follows. We state the major results in the next section. Section 3 contains a summary of facts about forward homomorphisms and Thom classes that we need. These results are surely well known; but I have not been able to find a suitable reference. I have stated the results about forward homomorphisms in slightly greater generality than we need. The fourth section contains some calculations of intersection numbers. The results concerning the generating varieties are proved in Section 5. The last section contains the proof of the main theorem and the obligatory list of the first few explicit formulas for