

# *On The Uniqueness in The Inverse Conductivity Problem with One Measurement*

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**Introduction.** We shall consider an inverse problem for electrically conductive material occupying a domain  $\Omega$  in  $\mathbb{R}^n$ ,  $n \geq 2$ . Let  $D$  be a subdomain of  $\Omega$  and suppose that the conductivity coefficient of  $D$  is 2 and of  $\Omega \setminus D$  is 1. We wish to determine the location of  $D$  by injecting current with density  $g$  across  $\partial\Omega$  and measuring the voltage  $u$  on a portion  $\Gamma_0$  of the boundary  $\partial\Omega$ . The voltage  $u$  satisfies the refraction problem:

$$\begin{aligned} \operatorname{div}((1 + \chi(D))\nabla u) &= 0 \quad \text{in } \Omega, \\ \frac{\partial u}{\partial N} &= g \quad \text{on } \partial\Omega \quad (N = \text{normal to } \partial\Omega) \end{aligned}$$

where  $\chi(D)$  is the indicator function of  $D$ , and  $u$  is normalized by

$$\int_{\partial_0\Omega} u = 0$$

where  $\partial_0\Omega$  is a compact subset of  $\partial\Omega$ . Thus the problem is to determine  $D$  either by one measurement, i.e., by using one specific choice of  $g$ , or perhaps by a finite number of measurements  $g_1, g_2, \dots, g_N$ .

If any number of measurements is allowed or, more precisely, if the Dirichlet-to-Neumann operator

$$u \Big|_{\partial\Omega} \longrightarrow \frac{\partial u}{\partial N} \Big|_{\partial\Omega}$$

is known for all functions  $u$  defined on  $\partial\Omega$ , then  $D$  is uniquely determined; this was proved, for  $\partial D$  piecewise analytic, by Kohn and Vogelius [6], and, for  $\partial D$  Lipschitz, by Isakov [5]; see also [4].

The case of a single measurement  $g$  was studied by Friedman and Gustafsson [2] and Bellout and Friedman [1]. They proved, roughly, that the local dependence of the data  $u|_{\Gamma_0}$  on the domain  $D$  is a function whose Frechet derivative does not vanish. Friedman and Vogelius [3] established a Lipschitz estimate for