

Uniqueness Criteria for Viscosity Solutions of Fully Nonlinear Elliptic Partial Differential Equations

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0. Introduction. This paper presents an extension of the results in [7] and [8] on the uniqueness of viscosity solutions of nonlinear elliptic partial differential equations. A viscosity solution is a type of weak solution first introduced in 1983 by M. G. Crandall and P.-L. Lions [2] for fully nonlinear first order partial differential equations. The concept of viscosity solution is motivated by the classical maximum principle which distinguishes it from other definitions of weak solution which usually involve integration by parts. P.-L. Lions [9] subsequently extended the original definition to include fully nonlinear, possibly degenerate, elliptic partial differential equations or FNPDE's for short.

Intuitively, a viscosity solution of the FNPDE

$$(0.1) \quad F(x, u(x), Du(x), D^2(x)) = 0 \quad \text{on } \Omega,$$

is a continuous function, $v : \Omega(\subset \mathbf{R}^n) \rightarrow \mathbf{R}$, such that the weak maximum principle holds when comparing $v(x)$ to any *classical* subsolution or supersolution of (0.1) on any open *subset* of Ω .

The function, F , in (0.1) is a continuous map

$$F : \Omega \times \mathbf{R} \times \mathbf{R}^n \times S(n) \rightarrow \mathbf{R},$$

where $S(n)$ is the set of real $n \times n$ symmetric matrices.

We impose the following two basic assumptions on F :

- $F(x, t, p, M) \geq F(x, t, p, N)$ for all (x, t, p, M, N) in $\Omega \times \mathbf{R} \times \mathbf{R}^n \times S(n) \times S(n)$ such that $(M - N)$ is positive semidefinite, i.e., F is degenerate elliptic.
- $F(x, t, p, M) \geq F(x, s, p, M)$ for all (x, t, s, p, M) in $\Omega \times \mathbf{R} \times \mathbf{R} \times \mathbf{R}^n \times S(n)$ such that $(s - t)$ is nonnegative, i.e., F is decreasing in $u(x)$.

We recall that a classical subsolution of (0.1) is a twice continuously differentiable function, $\Phi : \Omega \rightarrow \mathbf{R}$, such that

$$(0.2) \quad (F(x, \Phi(x), D\Phi(x), D^2\Phi(x)) \geq 0, \quad \text{for all } x \text{ in } \Omega,$$