

# Locally Continuous Operators, II

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**1. Introduction.** In this paper we consider bounded linear operators  $T \in \mathcal{B}(L_p[0,1], L_q[0,1])$  which satisfy special norm conditions when restricted to functions of small support. In particular, for any measurable set  $\Delta \subset [0,1]$  we denote by  $|\Delta|$  its Lebesgue measure and by  $P_\Delta$  the projection operator induced by the indicator function  $\chi_\Delta$ . Thus for  $f \in L_p[0,1]$ ,  $P_\Delta f = \chi_\Delta f$ .

**Definition 1.1.** Let  $\varphi(\cdot)$  and  $\psi(\cdot)$  be any two positive nondecreasing continuous functions on  $[0,1]$ . Let  $T \in \mathcal{B}(L_p[0,1], L_q[0,1])$ . We say that

(1)  $T$  is *left locally continuous* if for all measurable subsets  $\Delta \subset [0,1]$

$$(1.1) \quad \|TP_\Delta\|_{p \rightarrow q} \leq \varphi(|\Delta|);$$

(2)  $T$  is *right locally continuous* if for all measurable subsets  $\Delta \subset [0,1]$

$$(1.2) \quad \|P_\Delta T\|_{p \rightarrow q} \leq \psi(|\Delta|);$$

and

(3)  $T$  is *locally continuous* if for all measurable subsets  $\Delta, \Delta' \subset [0,1]$

$$(1.3) \quad \|P_{\Delta'} TP_\Delta\|_{p \rightarrow q} \leq \varphi(|\Delta|)\psi(|\Delta'|).$$

The terminology left, right, etc. is due to the result in Allen [1], Theorem 6, which for  $p = q = 2$  asserts that when in addition  $\varphi$  and  $\psi$  are continuous with  $\varphi(0) = \psi(0) = 0$ , then a unitary equivalent of  $T$  is

- (i) left locally continuous if and only if 0 is in the left essential spectrum of  $T$ ,
- (ii) right locally continuous if and only if 0 is in the right essential spectrum of  $T$ , and
- (iii) locally continuous if and only if 0 is in the reducing essential spectrum of  $T$ . See Salinas [17] for appropriate definitions.

In this paper we continue our study of these operators. In Section 2 we extend further the mapping properties of such operators extending the results in Allen [1] from the case when  $\varphi(s)$  and  $\psi(s)$  are powers ( $s^\alpha$ ) to more general functions. These type of results (for powers) have also been studied in a general context by Pisier [15] in connection with proving the equivalence of factorizations