

# Monge–Ampère Functions, II

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**Introduction.** The present article is a continuation of [F1]. It is dedicated to proving the following fundamental identity for Monge–Ampère functions  $f$ :

$$(\star) \quad \partial([df] \lrcorner \pi^* f) + [df] \lrcorner \alpha = 0.$$

The proof of  $(\star)$ , given in Section 2, is quite similar to that of Theorem 1.1, op. cit., except that the role played there by the “separation lemma” for integral currents is taken up by a “disintegration lemma” for normal currents (cf. Section 1). We note that if we use the notation of GMT, 4.3.1, or the theory of flat cochains (cf. [W]), then  $(\star)$  may be written

$$[df] \lrcorner \alpha - [df] \lrcorner \pi^* df = 0.$$

However, we prefer to use the form  $(\star)$ .

As an application of  $(\star)$  we derive some chain rules for compositions of  $C^{1,1}$  and Monge–Ampère functions. Other applications, with geometric implications, will be given in the third instalment of this series.

As mentioned in [F1], the identity  $(\star)$  is trivial in case  $[df]$  is the flat limit of a sequence  $[df_i]$ , where the  $f_i$  are smooth functions. It would be interesting to know whether this is true for all Monge–Ampère functions  $f$ . We suspect that a proof would rely on  $(\star)$ , which suspicion is part of our motivation for presenting this work publicly.

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## 0. Background.

**0.0.** Most of our notation and terminology may be found in [F1]. Here we give a few simple definitions and facts which may clarify parts of the text.

**0.1. Definition.** Let  $f : \mathbf{R}^m \rightarrow \mathbf{R}^n$  be locally lipschitzian. A *good approximation* to  $f$  is a sequence of smooth functions  $f_1, f_2, \dots : \mathbf{R}^m \rightarrow \mathbf{R}^n$  such that:

- (a)  $f_j \rightarrow f$  in the  $C^0$  topology;
- (b)  $\text{lip}(f_j|U) \leq C(U)$ ,  $j = 1, 2, \dots$  for each bounded open set  $U \in \mathbf{R}^m$ .
- (c)  $Df_j \rightarrow Df$  in  $L^1_{\text{loc}}(\mathbf{R}^m, \mathbf{R}^n)$ .