

# *The Inviscid Limit for the Navier–Stokes Equations of Compressible, Isentropic Flow with Shock Data*

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**ABSTRACT.** We solve the inviscid limit problem for the Navier–Stokes equations for one–dimensional, isentropic, compressible flow with shock data. We prove that the solution exists for all time, that it converges to the inviscid shock as the viscosity tends to zero, uniformly away from the shock, and that it converges uniformly in space to a travelling wave for fixed viscosity, as  $t$  tends to infinity. The overall analysis provides a very detailed picture of the qualitative properties of the viscous solution.

**1. Introduction.** An important problem in the theory of compressible fluid flow is to study the behavior of viscous flows as the viscosity becomes very small. In the special case that the underlying inviscid flow is smooth, the zero–dissipation limit question can be resolved using classical methods. However, inviscid, compressible flows contain shock waves in general; indeed, shock waves are fundamental in the theory of such flows. We therefore study here the zero–dissipation limit of a viscous flow for which the underlying inviscid flow is a single shock wave. Specifically, we prove global existence of solutions of the Navier–Stokes equations with shock data, and we show that the solution converges to the inviscid shock wave as the viscosity tends to zero, uniformly away from the shock. Thus, at the level of shock–wave solutions, the compressible Euler equations can be justified as the zero–dissipation limit of the Navier–Stokes equations.

Our analysis proves a very detailed picture of the qualitative properties of the solution. We show that the solution contains a convective discontinuity which decays exponentially in time and which vanishes in the zero–dissipation limit. For intermediate time we show that the solution is approximated well by solutions of the Burgers equation, appropriately lifted to phase space. And for large time we show that the solution coalesces with a viscous shock, which is a travelling wave solution of the Navier–Stokes equations.