

A Limiting Viscosity Approach for the Riemann Problem in Isentropic Gas Dynamics

MARSHALL SLEMROD & ATHANASIOS E. TZAVARAS

ABSTRACT. We consider the Riemann problem for the equations of isentropic gas dynamics in Eulerian coordinates. We construct solutions of this problem as limits of solutions of a “viscosity” regularized problem that is rigged so as to preserve the invariance of the original problem under dilatations of the independent variables. The solutions thus constructed may contain vacuum regions. Using the same approach, we also construct solutions of the Riemann problem in case the data contain a vacuum state.

1. Introduction. The equations describing one dimensional isentropic motions of a compressible, inviscid gas, written in spatial (Eulerian) coordinates and in conservative form, are

$$(1.1) \quad \rho_t + (\rho u)_x = 0 \quad -\infty < x < \infty, t > 0$$

$$(1.2) \quad (\rho u)_t + (\rho u^2 + p(\rho))_x = 0$$

Here, $\rho = \rho(x, t)$ and $u = u(x, t)$ stand for the density and the velocity of the gas, respectively, while $m := \rho u$ describes the momentum flux in the direction of the flow. The density ρ is nonnegative; the regions in the physical space where $\rho = 0$ are identified with vacuum regions of the flow. The pressure p is assumed to be a smooth function of ρ satisfying

$$(1.3) \quad p'(\rho) > 0 \text{ for } \rho > 0.$$

Under (1.3), Equations (1.1) and (1.2) form a hyperbolic system of conservation laws. The characteristic speeds $\lambda_{1,2} = u \mp \sqrt{p'(\rho)}$ are distinct when $\rho > 0$, but may coalesce when $\rho = 0$. The problem of solving (1.1)–(1.2) subject to initial data

$$(1.4) \quad (\rho(x, 0), u(x, 0)) = \begin{cases} (\rho_-, u_-), & x < 0 \\ (\rho_+, u_+), & x > 0 \end{cases}$$