

Nontangential Limits of Functions in Some $P^2(\mu)$ Spaces

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ABSTRACT. Denote area measure on the unit disk by A and Lebesgue measure on the unit circle by m . Let $d\mu = G dA + \omega dm$ where $\omega \in L^1_+(m)$ and where $G \in L^1_+(A)$ satisfies conditions shown by Kriete and MacCluer to guarantee that if I is an arc with ω log-integrable on I , $P^2(\mu|_D + \mu|_I)$ does not contain $L^2(\mu|_I)$ as a summand. If ω is log-integrable on some open subarc Γ of the unit circle, then for each $f \in P^2(\mu)$, $f(w) \rightarrow f(z)$ as $w \rightarrow z$ nontangentially a.e. on Γ . Moreover, $S_\mu \in \mathbf{A}_1 \setminus \mathbf{A}_2$ where \mathbf{A}_n is the class of operators introduced by Bercovici, Foias and Percy.

Introduction. Let μ be a positive, compactly supported regular Borel measure in the complex plane, \mathbf{C} , such that $P^\infty(\mu) = H^\infty(D)$. That is to say, $P^\infty(\mu)$, the weak-* closure of the polynomials in $L^\infty(\mu)$, is isometrically isomorphic and weak-* homeomorphic to the algebra of bounded analytic functions on the unit disk D . Let $P^2(\mu)$ denote the closure of the polynomials in $L^2(\mu)$ and S_μ , the operator on $P^2(\mu)$ defined by $(S_\mu f)(z) = zf(z)$, $f \in P^2(\mu)$.

For such a measure μ , the operator S_μ is, in some sense, the canonical subnormal operator. If S is subnormal and has a cyclic vector, then S is unitarily equivalent to S_ν for some positive, compactly supported regular Borel measure ν ; if S is irreducible, i.e., if S has no nontrivial reducing subspaces, then $\nu = \mu \circ \varphi^{-1}$, where μ is as above, and φ is a one-to-one, conformal mapping of the disk and a weak-* generator of $H^\infty(D)$. For details, see [2].

Olin and Thomson [6] have shown that if $P^\infty(\mu) = H^\infty(D)$ and if $P^2(\mu)$ has bounded point evaluations in an "outer hole" U in the support of μ , then each function $f \in L^\infty(\mu) \cap P^2(\mu)$ has the property that $\lim_{r \rightarrow 1^-} f(re^{i\vartheta}) = f(e^{i\vartheta})$ for almost every $e^{i\vartheta}$ in the strong boundary of U . Jim Thomson [7] has recently shown that if S_μ is irreducible then there is a simply connected region U in D with $\bar{U} \supset \partial D$ such that each f in $P^2(\mu)$ is analytic on U . A natural question is whether functions in $P^2(\mu)$ have nontangential limits $\mu|_{\partial U}$ almost everywhere.