

# *Local Norm Inequalities for Lacunary Series*

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ABSTRACT. We prove some norm inequalities for lacunary trigonometric and Walsh series.

It is known that for lacunary trigonometric series  $f(x) = \sum c_k e^{in_k x}$  with  $n_{k+1}/n_k \geq q > 1$ , its  $L^2$  norm over  $[0, 2\pi]$  is equivalent to its  $L^p$  norm for any  $0 < p < \infty$ . It is also known (see [4]) that for any measurable set  $E \subset [0, 1]$ ,  $|E| > 0$ , there exists  $N = N(E)$  so that if  $n_0 = 0, n_1 \geq N$ ,

$$\frac{1}{|E|} \int_E \left| \sum_{k=0}^{\infty} c_k e^{in_k x} \right|^2 dx \sim \sum_{k=0}^{\infty} |c_k|^2.$$

A similar result holds for Rademacher series, and this result has been generalized to  $0 < p < \infty$  in [2]. Rademacher series can be viewed as particularly nice lacunary Walsh series. Our purpose in this note is to generalize the result of [2] to lacunary series both in the context of Walsh and trigonometric functions.

The Rademacher functions are defined as follows:

$$\varphi_0(x) = \begin{cases} 1 & 0 \leq x < \frac{1}{2} \\ -1 & \frac{1}{2} \leq x < 1 \end{cases},$$

$$\varphi_0(x+1) = \varphi_0(x), \quad \text{and}$$

$$\varphi_k(x) = \varphi_0(2^k x), \quad k = 1, 2, \dots$$

The Walsh functions, in Paley's ordering, are defined as

$$w_0(x) = 1, \quad w_n(x) = \varphi_{a_1}(x) \varphi_{a_2}(x) \cdots \varphi_{a_\ell}(x),$$

where  $n = 2^{a_1} + 2^{a_2} + \cdots + 2^{a_\ell}$  with integers  $a_1 > a_2 > \cdots a_\ell \geq 0$ .