

Weak Resolvents of Linear Operators, II

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*To the memory of Allen Shields,
an inspirational mathematician and a wonderful human being*

ABSTRACT. The weak resolvent set of a bounded linear operator, a collection of analytic functions associated with the operator, was introduced in a previous paper. Here it is shown that operators with the same weak resolvent sets generate isomorphic uniformly closed algebras. In particular, this implies that strictly cyclic operators are similar if their weak resolvent sets are the same. The weak resolvent set is related to the polynomially convex hull of the spectrum, and a result on K -spectral sets is obtained. A study of joint weak resolvent sets of several operators is begun.

1. Introduction. The systematic study of weak resolvents of linear operators was begun in [11]. The set of all weak resolvents of an operator A (i.e., the set $\text{WR}(A)$ of all functions of the form $z \rightarrow G((z - A)^{-1}f)$ for $f \in \mathcal{X}$ and $G \in \mathcal{X}^*$) was shown to contain information about properties of A under various hypotheses.

We continue this investigation. We prove that A and B generate isomorphic uniformly closed algebras if $\text{WR}(A) = \text{WR}(B)$ (Corollary 2.3). This implies that B is similar to a self-adjoint operator if $\text{WR}(B)$ is contained in some $\text{WR}(A)$ for A self-adjoint (Corollary 2.5). We show that the behavior of elements of $\text{WR}(A)$ for large z determines the polynomially convex hull of $\sigma(A)$ (Theorem 3.3). It follows from a result on K -spectral sets (Theorem 3.7), and Stampfli's extension [12] of work of Brown [2], Brown, Chevreau, Pearcy [3] and Agler [1], that an operator has a nontrivial invariant subspace if its weak resolvent set is contained in that of a subnormal operator (Corollary 3.8).