

Complex Projective Surfaces of Nonnegative Kodaira Dimension Polarized by an Ample and Spanned Line Bundle of Genus Four

ANTONIO LANTERI & MARINO PALLESCHI

ABSTRACT. Let X be a complex projective smooth surface endowed with an ample line bundle L , which is spanned by its global sections. Pairs (X, L) as above are classified under the assumption that X is not birationally ruled and that $g(L) = 4$. The result is obtained by combining the geometry of the finite morphism associated to $|L|$ with the abstract structure of X .

Introduction. Let X be a smooth complex projective surface polarized by an ample line bundle L spanned by its global sections and let $g = g(L)$ be the genus of L . Pairs (X, L) as above with $g \leq 3$ are classified ([LP], [LL]), and here we deal with case $g = 4$. This study can be viewed as one of the ways to generalize the classification results of projective surfaces by sectional genus (e.g., [Io], [Li]). Actually, other different approaches were given by combining different assumptions on L with singularities of X , even in higher dimensions ([BLP], [BS1], [BS2], [F], [Ma]).

We only consider surfaces of Kodaira dimension $\kappa(X) \geq 0$. Our study proceeds by combining the geometry of the finite morphism $\Phi : X \rightarrow S = \Phi(X)$ associated to $|L|$ with the abstract structure of X . In particular, we describe X through its minimal model X' and the finite set F of points of X' one has to blow-up to get X . We also find the restrictions that F has to satisfy for L to be ample. Moreover, we associate to L an ample bundle L' on X' and we show that in most cases (X', L') is just the reduction of (X, L) . The set F has to satisfy some superabundance conditions with respect to L' in order to insure the spannedness of L . These conditions are handleable due to the assumption $\kappa(X) \geq 0$, which always makes the cardinality r of F very small. Such conditions would be far more complicated for ruled surfaces (e.g., see [LL] in case $g = 3$).

Let us briefly sketch our results. Let $d := L^2 = c_1(L)^2$; first of all we have $2 \leq d \leq 6$.